Apportionment Problems

Lecture 19
Section 4.1

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Hampden-Sydney College

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Outline

Apportioning Candies

Example

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- The score of 96 earns $96 \div 8.51 = 11.28$ candies.

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- The Senate divides 100 seats equally among the 50 states.

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2. No Person shall be a Representative who shall not have attained to the Age of twenty five Years, and been seven Years a Citizen of the United States, and who shall not, when elected, be an Inhabitant of that State in which he shall be chosen.
3. Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons.

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3. (continued) The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative;

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5. The House of Representatives shall chuse their Speaker and other Officers; and shall have the sole Power of Impeachment.

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- For example, if State A has twice the population of State B, then State A should have twice as many seats as State B.

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- How many seats should each state get?

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- How many seats should each state get?
- $326{,}766{,}748 \div 435 = 751{,}188$.
- Each seat should represent 751,188 people.

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- What to do?

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  - Calculate the exact number of seats that each state deserves, based on its population.
  - Separate each of those numbers into a whole number and the fractional part.
  - Give each state its whole number of seats.
  - Distribute the remaining seats to those states with the largest fractional parts.

Apply Hamilton's Method

Example

State	Population	Exact no. of Seats	Whole Numbers	No. of Seats Apportioned
VA	8,525,660			
MD	6,079,602			
PA	12,823,989			
DE	971,180			
Total				

- Apply Hamilton's method to the 4 states VA, MD, PA, DE.
- These four state currently have 11, 8, 18, and 1 seats, respectively, for a total of 38 seats.
- Each seat represents

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- So, Washington vetoed the bill.
- Congress failed to override Washington's veto.
- What happened next?

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- Chapter 4: Exercises 11, 12, 13, 14. Skip the terminology; apply Hamilton's method.